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Reply

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Capt. O'Neill and I are indebted to Mr. Yamagishi for his above comments pointing out the error in our paper [3]. Indeed the α_{opt} values from the theory as given in table 1 of his note should replace those given in table 3 of our paper. I must accept the responsibility for this error. After receiving Mr. Yamagishi's correspondence I traced this error to the use of an incorrect value of f_0^2 in the expression for K_{ijp}.

As pointed out by Yamagishi and as seen in his table 1, it appears that our observed α_{opt} values agree well with the theory as developed by Miyakoda [1] even for the three-dimensional (3-D) case. However, some caution must be exerted here. Miyakoda's analysis is developed for K_{ijp} constant and in the cases considered by Stuart and O'Neill [3] (i.e., the quasi-geostrophic omega equation) K_{ijp} varies with pressure due to the variation of static stability ($\sigma=-(\alpha/\theta)\partial\theta/\partial p$). Focusing only on the 2° grid case with $N_x=N_y=18$, $N_p=6$ and using σ for the standard atmosphere, the correct α_{opt} via the theory is: $\alpha_{opt}=0.237$ for $\sigma=1.178$ MTS units (at 800 mb.), $\alpha_{opt}=0.306$ for $\sigma=2.0$ MTS units (at 600 mb.), $\alpha_{opt}=0.352$ for $\sigma=4.0$ MTS units (at 440 mb.) and $\alpha_{opt}=0.414$ for $\sigma=44$ MTS units (at 200 mb.). Table 3 of Stuart and O'Neill [3] shows this case to have an observed $\alpha_{opt}=0.320$ with a sharp cutoff near $\alpha=0.350$. Hence, for the relaxation scheme used by Stuart and O'Neill for the solution of the 3-D omega equation a choice of $\sigma>4$ MTS units would have yielded an α_{opt} that led to nonconvergence. In this case the choice of the appropriate σ probably is not too difficult since only one level—200 mb.—had a very high σ and the tropospheric σ could be easily argued to be the most appropriate to yield a theoretical α_{opt} quite close to the observed α_{opt}. Actually the observed α_{opt} falls in the range of Miyakoda's theoretical α_{opt} corresponding to the range of σ but with a weighting toward the lower α's since more levels have lower σ's.

In an earlier correspondence (Stuart [2]), I reported on the extension of the model for the 2° grid case to $N_p=11$ (i.e. $\Delta p=10$ cb.) yet with $N_x=N_y=18$ as before. Miyakoda's theory gives the following values for the optimum over-relaxation factor: $\alpha_{opt}=0.128$ for $\sigma=0.944$ MTS units, $\alpha_{opt}=0.202$ for $\sigma=2.0$ MTS units, $\alpha_{opt}=0.272$ for $\sigma=4.0$ MTS units, and $\alpha_{opt}=0.418$ for $\sigma=206$ MTS units. (The first and last values correspond to σ at 900 and 100 mb. in the standard atmosphere.) In the actual solution of the omega equation, σ for the standard atmosphere was employed at all levels yielding an observed $\alpha_{opt}=0.15$ with a sharp cutoff near $\sigma=0.20$. Again we see that α_{opt} calculated via Miyakoda's theory has a wide variation depending on the σ value but our relaxation scheme yields an observed α_{opt} well within this variation and heavily weighted toward the lower (tropospheric) σ values. Note that nonconvergence would have occurred if we used an α_{opt} based on $\sigma>2$ MTS units.

The above comments and the observed results presented by Stuart and O'Neill [3] suggest some changes in our earlier conclusions concerning the α_{opt} value of the 3-D omega equation. Our observed optimum over-relaxation factors agree better with the limited theory than first thought and we now must definitely conclude that Miyakoda's [1] limited analysis is quite useful for selecting the range of α_{opt} for the 3-D omega equation. Since Miyakoda's theory shows α_{opt} to be quite sensitive to the stability factor, σ, in the 3-D omega equation, it is suggested to choose α_{opt} on the low side of the range of Miyakoda's theoretical α_{opt} values as determined using the range of σ appropriate to the problem. For some σ values the α_{opt} as determined by Miyakoda's analysis may lead to nonconvergence when employed in the quasi-geostrophic omega equation. Finally, the observed sharp cutoff for α just larger than α_{opt} is still an important feature of our observed α curves.

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